Simulink® Control Design

For Use with Simulink®

- Modeling
- Simulation
- Implementation

Advanced Topics

Version 1

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Simulink Control Design Advanced Topics

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1 Understanding and Controlling Results

1

Understanding and Controlling Results

To create accurate linearized models, it is important to be able to interpret the results and to understand the linearization algorithms. One method of interpreting the results is by simulating the linearized model and comparing the output with the original model. The linearization algorithms can be adjusted in various ways to control these results, as outlined in this chapter.

Comparing the Linearized and Original Models

Comparing simulations of the original model with simulations of the linearized model helps to determine if the linearized system behaves in a similar way to the original model. To make this comparison, re-insert the linearized subsystem into the model, configure the inputs and operating points so that they are the same as in the original model, and then compare output signals from a simulation of the two models.

When comparing models, remember that the states, inputs, and outputs of the linearized model are defined about an operating point of the original model, using the following variables:

$$\delta x(t) = x(t) - x_0$$
$$\delta u(t) = u(t) - u_0$$
$$\delta y(t) = y(t) - y_0$$

This means that when the original model is at the operating point $x(t)=x_0$, $u(t)=u_0$, $y(t)=y_0$, the linearized model will be at the operating point $\delta x(t)=0$, $\delta u(t)=0$, $\delta y(t)=0$. To compare the models accurately, subtract u_0 from input values and x_0 from the initial state values in the linearized model, then add y_0 to the output signal.

When you linearize only a portion of the original model, you should simulate the linearized model by substituting it back into the model in place of the original portion. This ensures that the operating point and inputs to the linearized portion are correct. To do this, export the linearized model to the workspace, delete the original portion from the model, and replace it with an LTI System block based on the linearized model.

Example

This example compares the `magball` model with the linearized model computed in "Linearizing the Model":

1. If you have not done so already, linearize the `magball` model at the targeted operating point computed in "Computing Operating Points from Specifications".

2. To create a new model containing the linearized plant system, first export the linearized model and operating point from the Control and Estimation Tools Manager to the MATLAB® workspace. To do this, right click the linearized model name in the project tree of the Control and Estimation Tools Manager. Select **Export** from the menu. Accept the default name for the model, `Model_sys`, and for the operating point, `Model_op`.

   Then, create a new Simulink® model, `magball_lin`, which is a copy of the original model, `magball`. Replace the Magnetic Ball Plant subsystem in `magball_lin` with an LTI System block (located in the Control System Toolbox category of the Simulink Library Browser). Import the linearized model into this block by entering `Model_sys` in the **LTI system variable** field in the Block Parameters window.

3. Set the operating points of the models by specifying the initial values of the states in the models:

   a. To set the initial values for `magball`, first enter the following at the command line

      ```
      [x,u]=getxu(Model_op)
      ```

      This returns vectors of state values and input values from the object `Model_op`.

      ```
      x =
        0.0500
        0
       -0.0000
        7.0036
        0
      u =
        []
      ```

      The ordering of states in these vectors is the same as that used in the Simulink model. To use the values in the state vector, `x`, as initial values for the model, select **Simulation -> Configuration Parameters** in the `magball` model window, then select the **Data Import/Export** panel. Select the check box next to **Initial State** and enter `x` on the right. Click **OK**.

b In `magball_lin`, the operating point values for the linearized system will all be zero since this subsystem was linearized about the operating point values. The operating point values in the Controller will be the same as in the original model since the Controller was not linearized. To create a vector of initial state values with the correct state ordering, first create a new operating point object for the system by typing

```
op=operpoint('magball_lin')
```

Change the operating point for the Controller in `op` to be the same as those in `Model_op`.

```
op.States(1).x=Model_op.States(1).x
```

This returns the following operating point.

```
Operating Point for the Model magball_lin.
(Time-Varying Components Evaluated at time t=0)

States:
----------
(1.) magball_lin/Controller/Controller
      x: 0
      x: -2.56e-006
(2.) magball_lin/LTI System/Internal
      x: 0
      x: 0
      x: 0

Inputs: None
```

Keep the operating point for the LTI system as zero. Extract vectors of states and inputs from this edited operating point.

```
[x1,u1]=getxu(op)
```

which returns

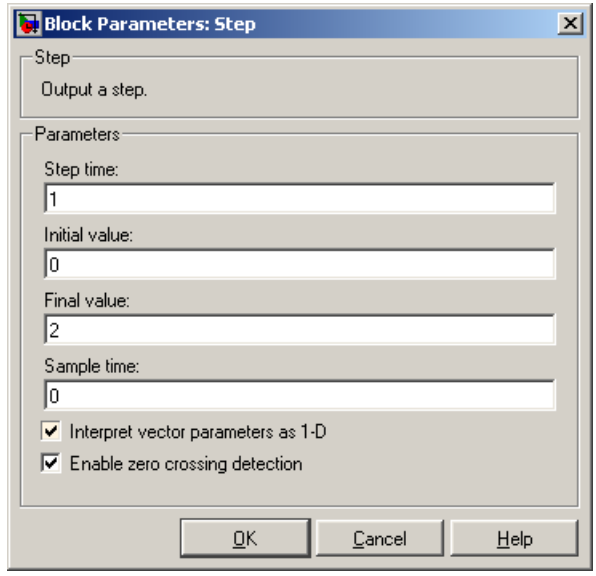
```
x1 =
  1.0e-005 *
         0
         0
         0
         0
   -0.2556
```

```
u1 =
     []
```

c To use the values in the state vector, `x1`, as initial values for `magball_lin`, select **Simulation -> Configuration Parameters** in the `magball_lin` model window, then select the **Data I/O** panel. Select the check box next to **Initial State** and enter `x1` on the right. Click **OK**

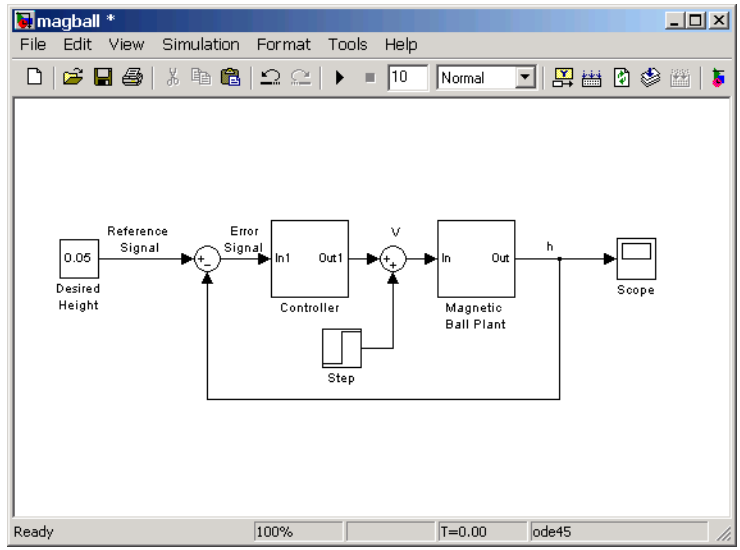
4 The output of `magball_lin` will be zero at the operating point. To create an output signal that is comparable with that in `magball`, add a Constant block, with a value of `0.05` to the output of `magball_lin`. Similarly, the input to `magball_lin` should be zero at the operating point. This is achieved by subtracting a value of 14 from the input signal of the linearized system. The operating point values, 0.05 and 14, were found using a Scope block to measure steady-state signal levels in the original model.

5 To observe the response of the models to a perturbation, add a Step block with the following parameter values to the input to the plant in both models.

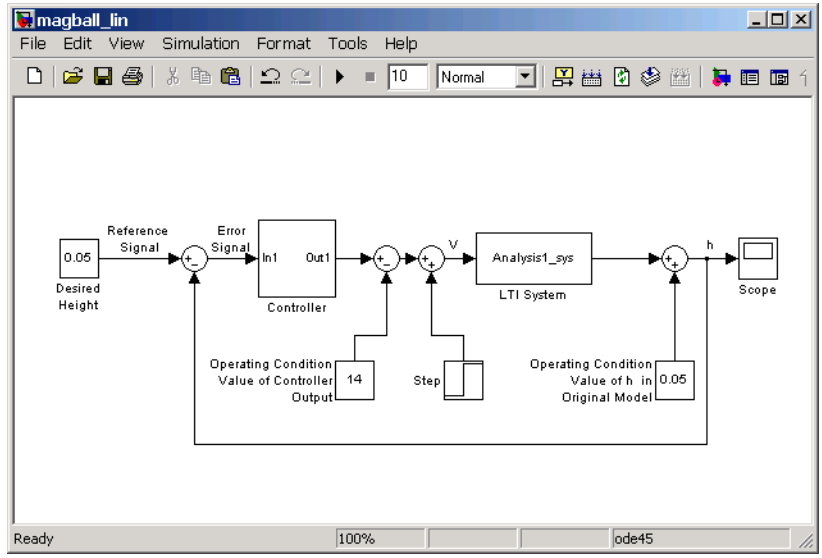


Parameter Values for Step Block

The model diagrams should now look like those in the following figures.

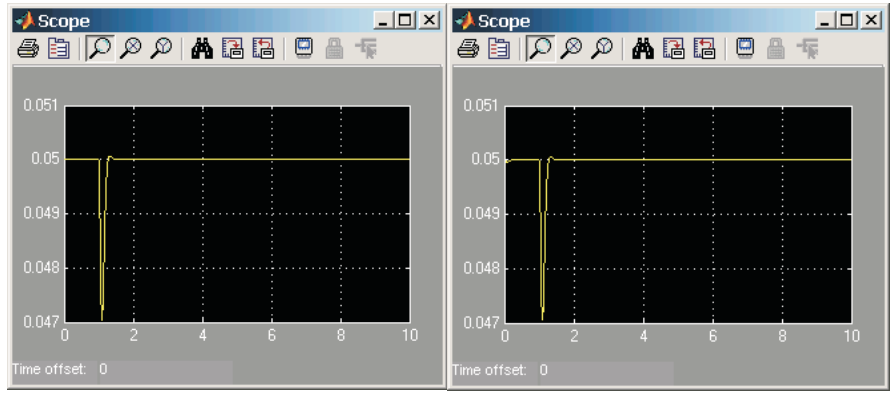


Magball Model with a Step Block Added to the Input



Magball Model with Linearized Magnetic Ball Plant

6 Run simulations in both models. The output signals, in the Scope blocks, are shown in the following figure.



Scope Blocks from Original (left) and Linearized (right) Models

As shown in the figure, both the original and linearized models react to the step input in a similar way.

Linearization Algorithms

Simulink Control Design can use two different linearization methods. The default method, which is used unless an option is selected, is called block-by-block analytic linearization. To use the alternative method, numerical-perturbation linearization, you must select an option in the Linearization Options dialog box of the GUI, or if using functions, with the `linoptions` function. The remainder of this chapter describes the two linearization methods in more detail and provides suggestions for controlling the results to create more accurate linearized models.

The default linearization method, block-by-block analytic linearization, linearizes the blocks individually and then combines the results to produce the linearization of the whole system. This method has several advantages:

- It divides the linearization problem into several smaller, easier problems.
- It defines the system being linearized by input and output markers on the signal lines rather than root-level inport and outport blocks.
- It supports open loop analysis.
- You can control the linearization of each block by using an analytic linearization that is programmed into the block or by selecting a perturbation level for the block.

The main disadvantage of this method is that for large or complicated systems it might be slower than numerical-perturbation linearization.

Alternatively, numerical-perturbation linearization linearizes the *whole system* by numerically perturbing the system's inputs and states about the operating point. This method has the advantage that it is quick and simple, especially for large or complicated systems. However, there are also several disadvantages with this method:

- It relies on root-level inport and outport blocks to define the system being linearized.
- There is no support for open loop analysis.
- You have limited control over the perturbation levels for each block.
- It does not use any of the analytic, pre-programmed block linearizations.

- It is sensitive to scaling issues (models with large and small signal values).

"Block-by-Block Analytic Linearization" on page 1-11 and "Numerical-Perturbation Linearization" on page 1-27 discuss these methods further.

Block-by-Block Analytic Linearization

Block-by-block analytic linearization is the default linearization method in Simulink Control Design. In this method, each of the blocks within the linearization path is first linearized individually. The linearization of the whole system is then computed by combining these results using the algorithm discussed in . This approach breaks the problem into several smaller problems. The following section gives details of the methods used to linearize each block, with suggestions for controlling the linearizations to create more accurate linearized models.

Individual Block Linearization Methods

There are two methods that Simulink Control Design uses to linearize the individual blocks in a model. Each method has options that you can control to create accurate linearized models.

Analytic Linearization

Many Simulink blocks contain analytic Jacobians for exact linearization. When linearizing a system using block-by-block analytic linearization, you can use these analytic linearizations instead of numerically perturbing the block. This is especially useful for blocks that contain discontinuities and do not give good results using numerical perturbation.

The following table lists the Simulink blocks that contain analytic Jacobians for linearization. For more information see the reference page for each block.

Analytic Block Jacobians

Block	Analytic Jacobian (Y/N)	Notes
Continuous Library		
Derivative	Y	Allows control of the time constant for the filter constant

Analytic Block Jacobians (Continued)

Block	Analytic Jacobian (Y/N)	Notes
Integrator	Y	OIncludes option to exclude saturation and resets from linearization
State-Space	Y	
Transfer Fcn	Y	
Transport Delay	Y	Allows control of Padé order
Variable Transport Delay	Y	Allows control of Padé order
Zero-Pole	Y	
Discontinuities Library		
Backlash	N	
Coulomb and Viscous Friction	N	
Dead Zone	Y	Includes option to treat as gain when linearizing
Dead Zone Dynamic	Y	
Hit Crossing	N	
Quantizer	Y	Includes option to treat as gain when linearizing
Rate Limiter	Y	Includes option to treat as gain when linearizing
Rate Limiter Dynamic	N	
Relay	N	
Saturation	Y	Includes option to treat as gain when linearizing
Saturation Dynamic	N	

Analytic Block Jacobians (Continued)

Block	Analytic Jacobian (Y/N)	Notes
Wrap to Zero	N	
Discrete Library		
Difference	Y	
Discrete Derivative	N	
Discrete Filter	Y	
Discrete State-Space	Y	
Discrete Transfer Fcn	Y	
Discrete Zero-Pole	Y	
Discrete-Time Integrator	Y	Includes option to ignore saturation and resets during linearization. Jacobian not supported for non-double data types.
First-Order Hold	N	
Integer Delay	N	
Memory	Y	Linearizes to a gain of 1
Tapped Delay	N	
Transfer Fcn First Order	Y	
Transfer Fcn Lead or Lag	Y	
Transfer Fcn Real Zero	Y	
Unit Delay	Y	Jacobian does not support frame-based signals
Weighted Moving Average	N	
Zero-Order Hold	N	

Analytic Block Jacobians (Continued)

Block	Analytic Jacobian (Y/N)	Notes
Logic and Bit Operations Library		
Bit Clear	N	
Bit Set	N	
Bitwise Operator	N	
Combinatorial Logic	N	
Compare To Constant	N	
Compare To Zero	N	
Detect Change	N	
Detect Decrease	N	
Detect Fall Negative	N	
Detect Fall Nonpositive	N	
Detect Increase	N	
Detect Rise Nonnegative	N	
Detect Rise Positive	N	
Extract Bits	Y	
Interval Test	N	
Interval Test Dynamic	N	
Logical Operator	N	
Relational Operator	N	
Shift Arithmetic	N	
Lookup Tables Library		

Analytic Block Jacobians (Continued)

Block	Analytic Jacobian (Y/N)	Notes
Cosine	N	
Direct Lookup Table (n-D)	N	
Interpolation (n-D) using PreLookup	Y	
Lookup Table	N	
Lookup Table (2-D)	N	
Lookup Table (n-D)	N	
Lookup Table Dynamic	N	
PreLookup Index Search	Y	
Sine	N	
Math Operations Library		
Abs	Y	
Add	Y	
Algebraic Constraint	N	
Assignment	N	
Bias	Y	
Complex to Magnitude-Angle	N	
Complex to Real-Imag	N	
Divide	Y	
Dot Product	N	
Gain	Y	

Analytic Block Jacobians (Continued)

Block	Analytic Jacobian (Y/N)	Notes
Magnitude-Angle to Complex	N	
Math Function	N	
Matrix Concatenation	N	
MinMax	N	
MinMax Running Resettable	N	
Polynomial	N	
Product	Y	
Product of Elements	Y	
Real-Imag to Complex	N	
Reshape	N	
Rounding Function	N	
Sign	Y	Linearizes to `Inf` at zero, linearizes to zero otherwise
Sine Wave Function	N	
Slider Gain	Y	
Subtract	Y	
Sum	Y	
Sum of Elements	Y	
Trigonometric Function	N	
Unary Minus	N	
Weighted Sample Time Math	N	

Analytic Block Jacobians (Continued)

Block	Analytic Jacobian (Y/N)	Notes
Model Verification Library		
Assertion	N/A	Does not contain outputs
Check Discrete Gradient	N/A	Does not contain outputs
Check Dynamic Gap	N/A	Does not contain outputs
Check Dynamic Lower Bound	N/A	Does not contain outputs
Check Dynamic Range	N/A	Does not contain outputs
Check Dynamic Upper Bound	N/A	Does not contain outputs
Check Input Resolution	N/A	Does not contain outputs
Check Static Gap	N/A	Does not contain outputs
Check Static Lower Bound	N/A	Does not contain outputs
Check Static Range	N/A	Does not contain outputs
Check Static Upper Bound	N/A	Does not contain outputs
Model Wide Utilities Library		
Block Support Table	N/A	Does not contain outputs
DocBlock	N/A	Does not contain outputs
Model Info	N/A	Does not contain outputs
Time-Based Linearization	N/A	Does not contain outputs
Trigger-Based Linearization	N/A	Does not contain outputs

Analytic Block Jacobians (Continued)

Block	Analytic Jacobian (Y/N)	Notes
Ports and Subsystems Library		
Configurable Subsystem	N/A	Only the blocks within the subsystem are part of the linearization
Atomic Subsystem	N/A	Only the blocks within the subsystem are part of the linearization
CodeReuse Subsystem	N/A	Only the blocks within the subsystem are part of the linearization
Enable	N	
Enabled and Triggered Subsystem	N/A	Only the blocks within the subsystem are part of the linearization
Enabled Subsystem	N/A	Only the blocks within the subsystem are part of the linearization
For Iterator Subsystem	N/A	Only the blocks within the subsystem are part of the linearization
Function-Call Generator	N/A	Only the blocks within the subsystem are part of the linearization
Function-Call Subsystem	N/A	Only the blocks within the subsystem are part of the linearization
If	N	
If Action Subsystem	N/A	Only the blocks within the subsystem are part of the linearization
Inport	N/A	Does not contain outputs
Model	N	
Outport	N/A	Does not contain inputs
Subsystem	N/A	Only the blocks within the subsystem are part of the linearization

Analytic Block Jacobians (Continued)

Block	Analytic Jacobian (Y/N)	Notes
Switch Case	N	
Switch Case Action Subsystem	N/A	Only the blocks within the subsystem are part of the linearization
Trigger	N	
Triggered Subsystem	N/A	Only the blocks within the subsystem are part of the linearization
While Iterator Subsystem	N/A	Only the blocks within the subsystem are part of the linearization
Signal Attributes Library		
Data Type Conversion	Y	
Data Type Conversion Inherited	Y	
Data Type Duplicate	N/A	Does not contain outputs
Data Type Propagation	N/A	Does not contain outputs
Data Type Scaling Strip	Y	
IC	N	
Probe	N	
Rate Transition	Y	
Signal Conversion	Y	
Signal Specification	Y	
Weighted Sample Time	N	
Width	N	
Signal Routing Library		
Bus Assignment	Y	

Analytic Block Jacobians (Continued)

Block	Analytic Jacobian (Y/N)	Notes
Bus Creator	Y	
Bus Selector	Y	
Data Store Memory	N/A	Does not contain inputs or outputs
Data Store Read	Y	Linearizes to a gain of 1. Assumes that there is no delay between data store read and data store write.
Data Store Write	Y	Linearizes to a gain of 1. Assumes that there is no delay between data store read and data store write.
Demux	N/A	
Environment Controller	Y	
From	N/A	
Goto	N/A	
Goto Tag Visibility	N/A	
Index Vector	Y	
Manual Switch	Y	
Merge	N	
Multiport Switch	Y	
Mux	N/A	
Selector	Y	
Switch	Y	
Sources Library - N/A No Inputs		
Sinks Library - N/A No Outputs		
User Defined Functions Library		

Analytic Block Jacobians (Continued)

Block	Analytic Jacobian (Y/N)	Notes
Embedded MATLAB Function	N	
Fcn	N	
Level-2 M-File S-Function	N	
MATLAB Fcn	N	
S-Function	N	
S-Function Builder	N	
Additional Math and Discrete Library		
Fixed-Point State-Space	Y	
Transfer Fcn Direct Form II	N	
Transfer Fcn Direct Form II Time Varying	N	
Unit Delay Enabled	Y	
Unit Delay Enabled External IC	Y	
Unit Delay Enabled Resettable	Y	
Unit Delay Enabled Resettable External IC	Y	
Unit Delay External IC	Y	
Unit Delay Resettable	Y	
Unit Delay Resettable External IC	Y	

Analytic Block Jacobians (Continued)

Block	Analytic Jacobian (Y/N)	Notes
Unit Delay With Preview Enabled	Y	
Unit Delay With Preview Enabled Resettable	Y	
Unit Delay With Preview Enabled Resettable External RV	Y	
Unit Delay With Preview Resettable	Y	
Unit Delay With Preview Resettable External RV	Y	
Decrement Real World	Y	
Decrement Stored Integer	Y	
Decrement Time To Zero	Y	
Decrement To Zero	Y	
Increment Real World	Y	
Increment Stored Integer	Y	

Several of these blocks include options to control the linearization that you can adjust in the Block Parameters window. For example, you can change the order of the Padé approximation used in the Transport Delay block or select the **Treat as gain when linearizing** option in the Saturation block. The

Notes column of the table above gives details on blocks that include these options.

Note The preprogrammed, analytic block linearizations are only used in block-by-block analytic linearization. When using numerical-perturbation linearization, these blocks will be numerically perturbed along with the rest of the system.

Block Perturbation

When a preprogrammed block linearization cannot be used, Simulink Control Design will compute the block linearization by numerically perturbing the states and inputs of the block about the operating point of the block. As opposed to the numerical-perturbation linearization method, this perturbation is local and its propagation through the rest of the model is restricted.

The block perturbation algorithm involves introducing a small perturbation to the nonlinear block and measuring the response to this perturbation. Both the perturbation and the response are used to create the matrices in the linear state-space model of this block. Changing the size of the perturbations will change the resulting linearized model.

As described in "Linearization of Nonlinear Models", a nonlinear Simulink block can be written as a state-space system:

$$\dot{x}(t) = f(x(t), u(t), t)$$
$$y(t) = g(x(t), u(t), t)$$

In these equations, $x(t)$ represents the states of the block, $u(t)$ represents the inputs of the block, and $y(t)$ represents the outputs of the block.

A linearized model of this system is valid in a small region around the operating point $t=t_0$, $x(t_0)=x_0$, $u(t_0)=u_0$, and $y(t_0)=g(x_0,u_0,t_0)=y_0$. Subtracting the operating point values from the states, inputs, and outputs defines a set of variables centered about the operating point:

$$\begin{aligned}\delta x(t) &= x(t) - x_o \\ \delta u(t) &= u(t) - u_o \\ \delta y(t) &= y(t) - y_o\end{aligned}$$

The linearized model can be written in terms of these new variables and is usually valid when these variables are small, i.e. when the departure from the operating point is small:

$$\begin{aligned}\delta\dot{x}(t) &= A\delta x(t) + B\delta u(t) \\ \delta y(t) &= C\delta x(t) + D\delta u(t)\end{aligned}$$

The state-space matrices A, B, C, and D of this linearized model represent the Jacobians of the block, as defined in "Linearization of Nonlinear Models". To compute the matrices, the states and inputs are perturbed, one at a time, and the response of the system to this perturbation is measured by computing $\delta\dot{x}$ and δy. The perturbation and response are then used to compute the matrices in the following way

$$A(:,i) = \frac{\dot{x}|_{x_{p,i}} - \dot{x}_o}{x_{p,i} - x_o}, \qquad B(:,i) = \frac{\dot{x}|_{u_{p,i}} - \dot{x}_o}{u_{p,i} - u_o}$$

$$C(:,i) = \frac{y|_{x_{p,i}} - y_o}{x_{p,i} - x_o}, \qquad D(:,i) = \frac{y|_{u_{p,i}} - y_o}{u_{p,i} - u_o}$$

where

- $x_{p,i}$ is the state vector whose ith component is perturbed from the operating point value.
- x_o is the state vector at the operating point.
- $u_{p,i}$ is the input vector whose ith component is perturbed from the operating point value.
- u_o is the input vector at the operating point.
- $\dot{x}|_{x_{p,i}}$ is the value of $\dot{x}$ at $x_{p,i}$, u_o.

- $\dot{x}|_{u_{p,i}}$ is the value of $\dot{x}$ at $u_{p,i}$, x_o.
- $\dot{x}_o$ is the value of $\dot{x}$ at the operating point.
- $y|_{x_{p,i}}$ is the value of y at $x_{p,i}$, u_o.
- $y|_{u_{p,i}}$ is the value of y at $u_{p,i}$, x_o.
- y_o is the value of y at the operating point.

Linearized models of discrete-time or multi-rate blocks are computed in a similar way. See "Linearization of Discrete-Time Models" and "Linearization of Multi-Rate Models" for the equations of linearized discrete-time and multi-rate systems.

Note A perturbed value is one that has been changed by a very small amount from the operating point value. The default difference between the perturbed value and the operating point value is $10^{-5}(1+|x|)$ for block-by-block analytic linearization, where x is the operating point value.

Changing the size of the perturbations will change the linearization results. The default perturbation size is $10^{-5}(1+|x|)$ where x is the operating point value of the state or input being perturbed. To change the perturbation size of the states in the Magnetic Ball Plant block in the `magball` model to $10^{-7}(1+|x|)$, type

```
blockname='magball/Magnetic Ball Plant'
set_param(blockname,'StatePerturbationForJacobian','1e-7')
```

To change the perturbation size of the input of the Magnetic Ball Plant block to $10^{-7}(1+|u|)$, where u is the input signal level, follow these steps:

1 Get the block's port handles

```
ph=get_param('magball/Magnetic Ball Plant','PortHandles')
```

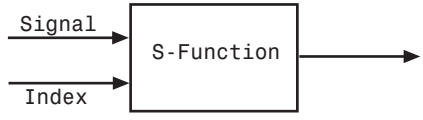
2 Get the inport

```
pin=ph.Inport(1)
```

3 Set the perturbation level for this inport

```
set_param(pin,'PerturbationForJacobian','1e-7')
```

If there is more than one inport, you can choose to assign a different perturbation level to each. The following figure shows an S-Function block with two input signals, the actual signal and an index variable. Since you probably do not want to perturb the index signal, you can assign a perturbation level of zero to this inport.



Block Containing Two Inports

Numerical-Perturbation Linearization

An alternative linearization method available for use in Simulink Control Design is numerical-perturbation linearization, which computes state-space matrices for the linearized model by numerical perturbation of the *whole system*. The method is relatively quick and simple, although as mentioned in "Linearization Algorithms" on page 1-9, it does have some disadvantages.

Numerical-perturbation linearization requires that root-level inport and outport blocks be present in the model. These blocks define the portion of the model that you want to linearize instead of inserting input and output points by right-clicking on the signal lines. Any input, output, or open loop points on signal lines in the model will be ignored when using numerical-perturbation linearization.

The perturbation is introduced to the system at the root level inport blocks and in the states of the system. The response to the perturbation is measured at the outport blocks.Suggestions for controlling the results of numerical-perturbation linearization to create accurate linearized models are given in "Controlling the Results of Numerical-Perturbation Linearization" on page 1-30

Invoking Numerical-Perturbation Linearization

Prior to Simulink 3.0, numerical-perturbation linearization was the only linearization method available with Simulink. Although block-by-block analytic linearization is now the default linearization method, you might choose to use numerical-perturbation linearization if your model is very large or complicated.

To use numerical-perturbation linearization with the Simulink Control Design GUI, select **Tools -> Options** while in the **Linearization Task** node of the Control and Estimation Tools Manager and select `Numerical-Perturbation` from the **Linearization Algorithms** menu.

To use numerical-perturbation linearization with the `linearize` function, set the `LinearizationAlgorithm` option to `'numericalpert'` with the `linoptions` function.

```
linopt=linoptions('LinearizationAlgorithm','numericalpert')
```

To linearize the model, type

```
sys=linearize('modelname',op,linopt)
```

where `modelname` is the name of the model being linearized and `op` is the operating point object for the system.

Perturbation Algorithm

The numerical perturbation algorithm involves introducing a small perturbation to the nonlinear model and measuring the response to this perturbation. Both the perturbation and the response are used to create the matrices in the linear state-space model. Changing the size of the perturbations will change the resulting linearized model.

As described in "Linearization of Nonlinear Models", a nonlinear Simulink model can be written as a state-space system:

$$\dot{x}(t) = f(x(t), u(t), t)$$
$$y(t) = g(x(t), u(t), t)$$

In these equations, $x(t)$ represents the states of the model, $u(t)$ represents the inputs of the model, and $y(t)$ represents the outputs of the model.

A linearized model of this system is valid in a small region around the operating point $t=t_0$, $x(t_0)=x_0$, $u(t_0)=u_0$, and $y(t_0)=g(x_0,u_0,t_0)=y_0$. Subtracting the operating point values from the states, inputs, and outputs defines a set of variables centered about the operating point:

$$\delta x(t) = x(t) - x_o$$
$$\delta u(t) = u(t) - u_o$$
$$\delta y(t) = y(t) - y_o$$

The linearized model can be written in terms of these new variables and is usually valid when these variables are small, i.e. when the departure from the operating point is small:

$$\begin{aligned}\delta\dot{x}(t) &= A\delta x(t) + B\delta u(t)\\ \delta y(t) &= C\delta x(t) + D\delta u(t)\end{aligned}$$

The state-space matrices A, B, C, and D of this linearized model represent the Jacobians of the system, as defined in "Linearization of Nonlinear Models". To compute the matrices, the states and inputs are perturbed, one at a time, and the response of the system to this perturbation is measured by computing $\delta\dot{x}$ and δy. The perturbation and response are then used to compute the matrices in the following way

$$A(:,i) = \frac{\dot{x}|_{x_{p,i}} - \dot{x}_o}{x_{p,i} - x_o}, \qquad B(:,i) = \frac{\dot{x}|_{u_{p,i}} - \dot{x}_o}{u_{p,i} - u_o}$$

$$C(:,i) = \frac{y|_{x_{p,i}} - y_o}{x_{p,i} - x_o}, \qquad D(:,i) = \frac{y|_{u_{p,i}} - y_o}{u_{p,i} - u_o}$$

where

- $x_{p,i}$ is the state vector whose ith component is perturbed from the operating point value.
- x_o is the state vector at the operating point.
- $u_{p,i}$ is the input vector whose ith component is perturbed from the operating point value.
- u_o is the input vector at the operating point.
- $\dot{x}|_{x_{p,i}}$ is the value of $\dot{x}$ at $x_{p,i}$, u_o.
- $\dot{x}|_{u_{p,i}}$ is the value of $\dot{x}$ at $u_{p,i}$, x_o.
- $\dot{x}_o$ is the value of $\dot{x}$ at the operating point.
- $y|_{x_{p,i}}$ is the value of y at $x_{p,i}$, u_o.

- $y|_{u_{p,i}}$ is the value of y at $u_{p,i}$, x_o.
- y_o is the value of y at the operating point.

Linearized models of discrete-time or multi-rate systems are computed in a similar way. See "Linearization of Discrete-Time Models" and "Linearization of Multi-Rate Models" for the equations of linearized discrete-time and multi-rate systems.

Note A perturbed value is one that has been changed by a very small amount from the operating point value. The default difference between the perturbed value and the operating point value is $10^{-5} + 10^{-8}|x|$ for numerical-perturbation linearization.

Controlling the Results of Numerical-Perturbation Linearization

Several factors influence the creation of accurate linearized models. "What Is Linearization?" discusses some of these factors, such as careful selection of operating points. Factors that are particular to numerical-perturbation linearization are presented here, with suggestions for controlling them.

Setting the Perturbation Level

In numerical-perturbation linearization, there are three options for setting the perturbation levels of states and inport blocks:

1. You can accept the default perturbation levels. The default perturbation levels for the states are $10^{-5} + 10^{-8}|x|$, where x is a vector of the operating point values for the states in the model. Similarly, default perturbation levels for the inport blocks are $10^{-5} + 10^{-8}|u|$, where u is a vector of the operating point values for the inputs in the model.
2. You can edit the linearization property `NumericalPertRel` using the `linoptions` function. The value of this property adjusts the perturbations in the following way:

- The perturbation of the states is $\text{NumericalPertRel} + 1e-3 \times \text{NumericalPertRel} \times |x|$.
- The perturbation of the inputs is $\text{NumericalPertRel} + 1e-3 \times \text{NumericalPertRel} \times |u|$.

3 You can provide vectors of perturbation levels for the states and inport blocks. These values override the values computed using the `NumericalPertRel value`. Specify the perturbation levels by editing the linearization properties `NumericalXPert` and `NumericalUPert` using the `linoptions` function. The properties `NumericalXPert` and `NumericalUPert` are vectors of absolute perturbation levels.

Handling Special Blocks

Certain blocks, especially those containing discontinuities such as `Saturation` or `Transport Delay`, may not linearize well using numerical-perturbation. Although these blocks often have preprogrammed linearizations that are used with block-by-block analytic linearization instead of numerically perturbing them, they are *not* used in numerical-perturbation linearization. An alternative solution is to replace these blocks with an appropriate block before linearizing your model. For example, you might choose to replace a `Saturation` block with a `Gain` block.

Handling Feedback Loops

"Understanding Open Loop Analysis" discusses the effect of feedback loops on the results of a linearization. With block-by-block analytic linearization, you can perform open loop analysis without removing feedback loops. When using numerical-perturbation linearization, the only way to remove the effect of feedback loops is to manually remove them from the model *and* manually force the operating point to remain the same as the original model.

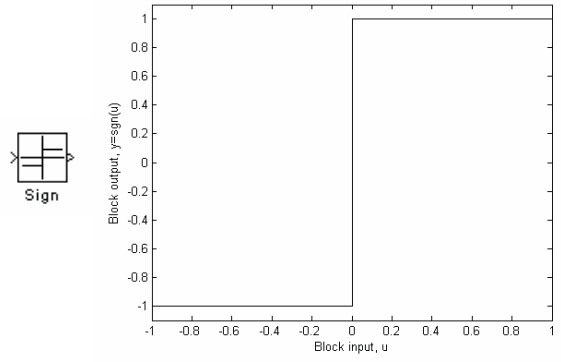
Recommendations for Creating Accurate Linearized Models

Particular blocks and modeling situations in Simulink can sometimes cause linearization difficulties. However, by understanding what it means to linearize a Simulink model and by using the correct modeling techniques, you can create accurate linearized models for use in further analysis and design.

This section consists of examples that highlight modeling situations that can lead to problems when computing linearized models, with recommendations for ways to avoid these situations. The examples focus on the following modeling situations:

Blocks with Discontinuities

There are several Simulink blocks that contain discontinuities, such as the Sign block, whose behavior is shown in the following figure.



The very large derivatives that occur at the point of discontinuity can cause problems with linearization. For example, the Sign block has the following linearization

$$D = 0, u \neq 0$$

$$D = \infty, u = 0$$

where D is a state-space matrix, and u is the input signal to the block.

When these blocks are within the linearization path of your model, the resulting linearized model could potentially have very large values. There is no obvious solution to this problem and it is recommended that you remove or replace these blocks. However, when your model operates in a region away from the point of discontinuity, the linearization will be zero. This should not cause any problems, although when the linearizations of several blocks are multiplied together (as in a feedback path) it can cause the linearization of the system to be zero.

When these blocks are outside the linearization path, they can still contribute to the definition of the operating point of the model but will not otherwise affect the linearization. It is safe to use them for reference signals, disturbances, and any other signals and blocks that are not being linearized.

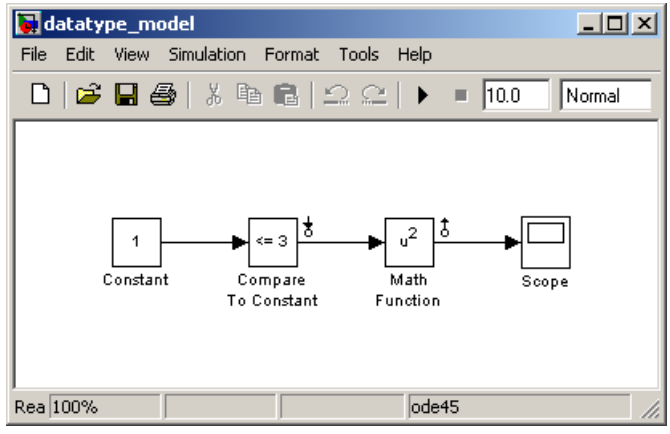
Other examples of blocks with discontinuities include

- Relational Operator blocks
- Relay block
- Logical Operator blocks
- Stateflow blocks
- Quantizer block (has an option to treat as a gain when linearizing)
- Saturation block (has an option to treat as a gain when linearizing)
- Deadzone block (has an option to treat as a gain when linearizing)

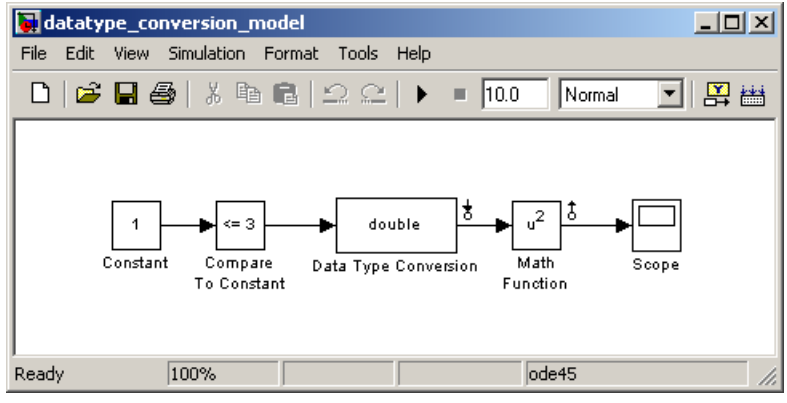
Non-Double Data Types

Blocks that have non-double data type signals as either inputs or outputs, and which do not have a preprogrammed exact linearization, will automatically linearize to zero as they cannot be numerically perturbed. For example, many logical operator blocks have Boolean outputs and will therefore linearize to zero.

To work around this problem, you can use a Data Type Conversion block, which does have a preprogrammed exact linearization, to convert your signals to doubles before linearizing the model. The following example illustrates this concept. The model in this example is configured to linearize the Square block at an operating point where the input is 1. The resulting linearized model should be 2 but the input to the Square block is Boolean and the linearization is zero.



However, by inserting a Data Type Conversion block before the linearization input point, the input signal to the Square block is a double, and the linearized model gives the correct response of 2.



Overriding Non-Double Data Types

When linearizing a model that contains non-double data types but still runs correctly in all double precision, you can choose to override all data types with doubles. To do this, in the model window select **Tools —> Fixed-Point Settings** from the menu. This opens the **Fixed-Point Settings** window. Within this window select **True doubles** from the **Data type override** menu. When linearizing and simulating the model, it now uses doubles for all data types.

Note This method does not work when the model relies on other data types in its algorithm, such as relying on integer data types to perform truncation from floats.
